THE MICHELSON INTERFEROMETER

INTRODUCTION

This interferometer was originally designed by A Michelson for the accurate spectral analysis of light sources. For example, using the instrument he was able to show that the red H_α Balmer line of the hydrogen spectrum was composed of two components separated by only 0.014 nm. He was also able to measure the natural width of the cadmium red line. Nowadays this instrument is normally used for the accurate determination of wavelengths, refractive indices of gases and transparent materials and small changes of length.

THEORY

The main features of the interferometer are shown schematically in Figure 1.

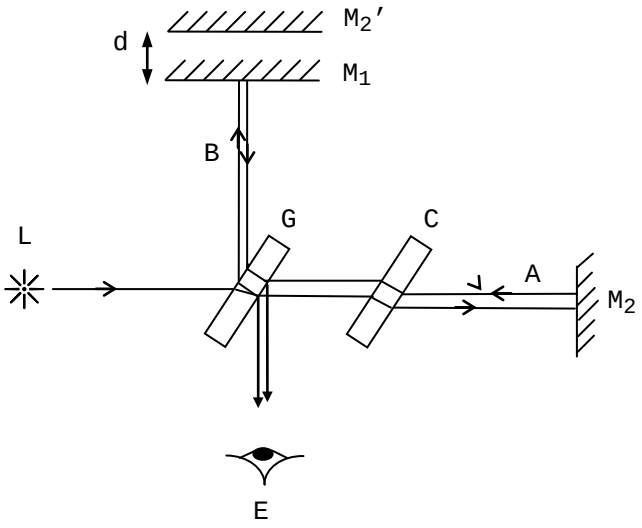

Fig. 1 The setup of the Michelson interferometer

M_1 and M_2 are two plane mirrors silvered on the front surfaces. They are mounted vertically on two rigid holders placed at the sides of a flat metal stand. Screws are provided in front of the holders, adjusting of which allows M_1 and M_2 to be tilted. M_1 can also be moved horizontally by a micrometer attached to the M_1 holder. G, the beam splitter, is a plane glass plate slightly silvered on one side. C is the compensator plane glass plate of the same thickness as G. Both are mounted vertically and at an angle 45° to the direction of the incident light.

When light from an extended light source is allowed to fall on G, one portion, calling it beam A, is transmitted through G and C to M_2 and the other, calling it beam B is reflected by G to M_1. Beam A, returning from M_2, is reflected at the back of G into the eye placed at E and beam B, after reflected from M_1 passes through G to reach the eye E.

We shall now see how the combination of these two portions of light which have started as one single beam and gone through different optical paths give rise to interference fringes. To do this we refer to Figure 2.

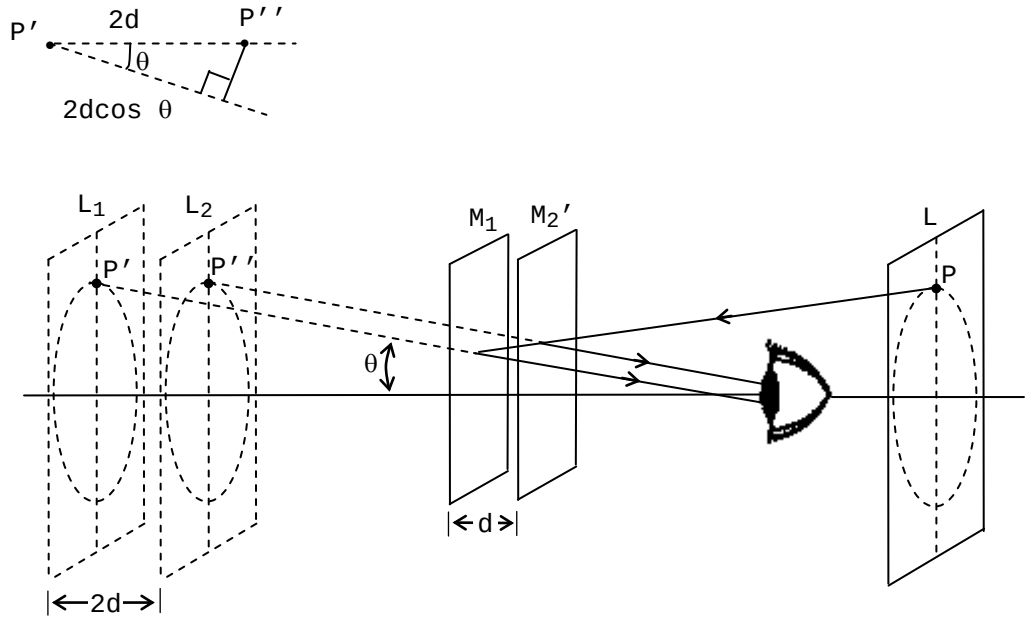

Fig. 2 Formation of fringes by Michelson interferometer

The mirror M_2 is here replaced by its virtual image M_2' formed by reflection in G. If M_1 and M_2 have been set vertical and perpendicular to one another, then M_2' is parallel to M_1. Owing to the mirror action of G, we may now think the light source as being the observer and as forming two virtual images L_1 and L_2 in M_1 and M_2' respectively. If d is the separation of M_1 and M_2', the virtual sources L_1 and L_2 will be separated by 2d.

For rays of light reflected normal to the mirrors, the phase difference due to the path difference is $4\pi d/\lambda$. Also, an additional phase shift of π is introduced because beam A is reflected off the outer side while beam B is reflected off the inner side of the beam splitter. Therefore, the total phase difference is

$$\delta = 4\pi d/\lambda - \pi \ .$$

If the light rays satisfy the conditions:

$$\delta = 2m\pi \ ,$$

or

$$2d = (m + ½)\lambda \qquad (1)$$

where m is an integer and λ the wavelength of incident light, they will be in phase. However, parallel rays of light reflected at an angle are generally not in phase. They will be in phase only for those angles θ that satisfy the relation

$$2d\cos\theta = (m + ½)\lambda \qquad (2)$$

On entering the eye that has been adjusted to receive parallel rays of light, these rays will reinforce each other to produce constructive interference resulting in maxima of light intensity at their focal points on the eye retina.

For those reflected rays of light satisfying

$$2d\cos\theta = m\lambda \qquad (3)$$

destructive interference will occur resulting in minima of light intensity at their focal points on the eye retina.

Since for fixed d and λ and different values of m, a system of dark and bright circle concentric fringes each corresponding to a constant θ will be observed, such interference fringes are therefore known as fringes of equal inclination. They are localized at infinity.

If the two mirrors are not exactly vertical and perpendicular to one another, then the space between M_1 and M_2' will be in the shape of a wedge, the fringes observed in this case will be straight and parallel. Such fringes are known as fringes of equal thickness. They are localized at finite distance.

EXPERIMENTS

i) To Calibrate the Micrometer Screw of the Interferometer

Place a Na light source at L and a cross-wire between L and G. View the cross-wire from the position E and adjust the tilting screws behind the mirror M_2 so as to superimpose the two images (one from M_1 and one from M_2). When the two images coincide, interference fringes will be visible. Further adjusting the tilting screws slightly at this stage will render the fringes circular.

Turn the micrometer screw until distinct and clear circular fringes are observed. Note down the micrometer screw reading and count the number of fringes that disappear (or appear) at the centre of the fringe system. Take 6 sets of readings each for the appearance/disappearance of 50 fringes and plot a graph of micrometer reading versus fringe number. Alternatively, you can tilt the screws behind M_2 to obtain straight-line fringes and take 6 sets of reading each for 50 fringes crossing the centre of the cross-wire.

From the slope of the graph and the average wavelength λ of the Na D-line (589.294 nm), determine the ratio (k) of the movement M_1 to the micrometer screw reading.

Since the number Δm of fringes appear/disappear corresponding to the movement of Δd of M_1 (as the result of the micrometer being turned through a distance ΔD) is given by

$$2\Delta d = (\Delta m)\lambda \qquad (4)$$

Therefore $$k = \Delta d/\Delta D = \lambda(\Delta m)/2(\Delta D) \qquad (5)$$

In the following experiments, the results of the conversion of the micrometer movement to that of M_1 will be required, hence it is important that the ratio k be determined accurately.

ii) To Determine the Absolute Value of Wavelength

Place a green filter in front of Hg light source.

Measure the wavelength of the Hg green line by setting up the circular fringes and count the number Δm of fringes appearing/disappearing as M_1 is moved slowly from position d_1 to position d_2. The wavelength is given by

$$2(d_1 - d_2) = (\Delta m)\lambda \qquad (6)$$

The distance moved by the M_1 i.e. (d_1 and d_2) = Δd is to be determined from the graph obtained in Experiment (i). Repeat the procedure five times each time for $\Delta m = 20$. Estimate the error in your measurement.

iii) To Measure the Na doublet Separation

The wavelength separation of the Na D-line doublet is easily determined by observing the successive coincidence and discordance of the two sets of fringe systems produced by the doublet of wavelengths (λ_1 and λ_2 with $\lambda_1 > \lambda_2$). As d is increased, the two systems gradually separate and the maximum discordance occurs when the rings of one system are set exactly halfway between those of the other system. The discordance positions are most clearly seen as minima in the contrast of the pattern.

For the first discordance, if we confine our attention to the rings at the centre ($\theta = 0$), equation (2) can be written as:

$$2d_1 = m_1\lambda_1 = (m_1 + ½)\lambda_2 \qquad (7)$$

On further increasing d, the rings will coincide and then separate again. At the next discordance, we have

$$2d_2 = m_2\lambda_1 = (m_2 + 1½)\lambda_2 \qquad (8)$$

Subtracting (7) from (8), we have

$$2(d_2 - d_1) = (m_2 - m_1)\lambda_1 = (m_2 - m_1 + 1)\lambda_2$$

$$(m_2 - m_1) = \lambda_2/(\lambda_1 - \lambda_2) \qquad (9)$$

From (9) and to 1st order approximation, we have

$$2(d_2 - d_1) = \frac{\lambda_2\lambda_1}{\lambda_1 - \lambda_2} \approx \frac{\lambda^2}{\lambda_1 - \lambda_2}$$

$$\lambda_1 - \lambda_2 = \frac{\lambda^2}{2(d_2 - d_1)}$$

where λ is the average wavelength.

Set up the instrument to give circular fringes. Turn the micrometer screw consistently in one direction and note the fringes pass alternately from a condition of high contrast to one of almost complete disappearance. Take the micrometer readings at positions of almost complete disappearance of fringes and draw a graph of readings against the nature numbers of consecutive fringe disappearance. From the slope of the graph and the ratio k, determine the wavelength difference of the Na doublet. Estimate the error of your result and compare your value with the accepted value.

iv) <u>To Determine the Refractive Index of a Thin transparent sheet</u>

Set up the interferometer to produce straight line fringes. To do this, first obtain circular fringes using Na light source and then turn the micrometer screw so that fewer and fewer rings are visible. When one large indistinct fringe occupies the entire field – this is the situation where M_2' coincides exactly with M_1, i.e. the two light beams received at the eye has zero optical path difference (OPD), tilt M_2 slightly (finely adjust the micrometer if necessary) until the fringes become straightest and sharpest.

Now substitute white light for the Na light and look for white light fringes. White light fringes are localized fringes with a dark fringe at the centre and 5 or 6 coloured fringes on both sides. Turn the micrometer slowly until white light fringes appear at the centre of the field of view. Note down the reading of the micrometer and then insert a transparent sheet in the beam GM_2. The transparent sheet increases the optical path of one of the light beams and hence destroys the zero OPD condition. As a result, white light fringes disappear. To get back to the zero OPD condition, turn the micrometer until the white light fringes return to the centre of the field of view. Note down again the reading of the micrometer.

If n is the number of Na fringes corresponding to the distance D moved by the micrometer to restore the OPD condition, then

$$2t\mu - 2t = n\lambda$$

or $$\mu = n\lambda/2t + 1$$

where t is the thickness of the transparent sheet and μ its refractive index.

Measure t with a micrometer gauge and obtaining n from graph of Experiment (i) and hence determine the refractive index of the transparent sheet. What material is the sheet made of?

QUESTIONS

1. Explain why the circular fringes change in size when the distance between M_1 and M_2' is changed.

2. Explain the formation of white light fringes and the reason for the central fringe being dark and not bright as observed in this experiment.

v) To Calibrate the Micrometer Screw of the Fabry–Pérot Interferometer

Place a sodium lamp at L. The lamp needs a warm-up time of approximately 20 minutes to reach steady output. Adjust the preset micrometer till the spacing between the two mirrors M_1 and M is approximately 1~2 mm.

Warning: ALWAYS keep an eye on the mirror gap while turning the screws to avoid the possibility of the mirrors touching.

Place the pinhole plate on the open window of the sodium lamp, observe these pinhole images generated by multiple reflections of the two mirrors behind the Fabry–Pérot cavity, and adjust the two kinematic screws of M_1 to superimpose all pinhole images.

Replace the pinhole plate with the ground glass (G) to create an extension light source. Be careful to avoid touching the hot lamp cover. Put a convex lens (f = 45 mm) at L_1, as shown in Figure 3. Adjust the distance between the lamp and lens and sharp multi-beam interference rings can be observed.

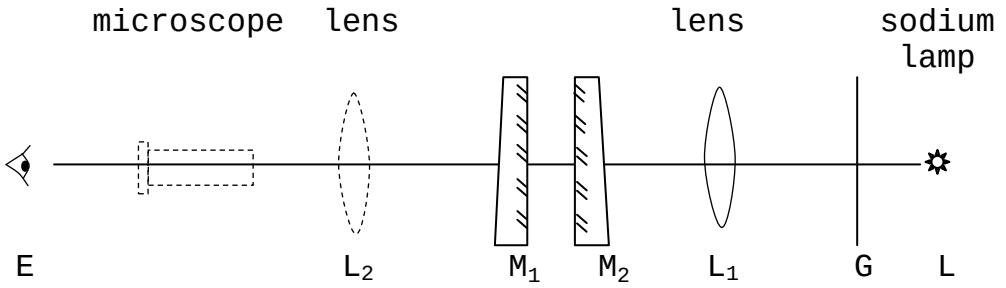

Fig. 3 Schematics of the Fabry–Pérot Interferometer.

Move your eye up and down to see whether the diameter of the central ring changes, finely adjust the right-upper tilting screw of M_1 mirror till no diameter changes are observed. Similarly, move your eye left and right, and adjust the left-lower screw. Finally, the two mirrors will be almost perfectly parallel when there are no diameter changes observed anywhere.

This experiment can be conducted by just observing behind the Fabry–Pérot cavity with the naked eye, but it would be better to use the provided convex lens (L_2) and the mini microscope as shown in Fig. 3.

Note down the fine micrometer screw reading. Turn the screw and count the number of fringes that disappear (or appear) at the centre of the fringe system. Take 6 sets of readings each for the appearance/disappearance of 50 fringes and plot a graph of micrometer reading versus fringe number. Alternatively, you can use the mini microscope and convex lens as shown in Fig. 3, tilt the screws behind M_2 to obtain straight-line fringes and take 6 sets of reading each for 50 fringes crossing the centre of the eyepiece of the microscope.

From the slope of the graph and the average wavelength λ of the Na D-line (589.294 nm), determine the ration (k) of the movement M_1 to the fine micrometer reading. Since the number Δm of fringes appear/disappear corresponding to the movement of Δd of M_1 (as the result of the micrometer being turned through a distance ΔD) is given by

$$2\Delta d = (\Delta m)\lambda$$

Therefore $k = \Delta d/\Delta D = \lambda(\Delta m)/2(\Delta D)$

In the following experiments, the results of the conversion of the micrometer movement to that of M_1 will be required, hence it is important that the ratio k be determined accurately.

vi) To determine Na doublet separation

Move the M_1 close to M_2 with a separation of approximately 1~2 mm by adjusting the preset micrometer. Each of the two sodium lines gives rise to an independent rings system. The movable mirror can be adjusted so that the two rings systems coincide or so that the two sets of rings can be seen separately. Move the fine micrometer screw to give either coincidence or anticoincidence of the ring systems and read the micrometer. Continue moving and reading the fine micrometer at successive coincidence or anticoincidence (i.e. when one set of rings is half way between the other set) and then plot a graph of micrometer readings against the natural numbers.

From the slope and the value of k, a conversion ratio, obtained in a separate experiment, find the distance d between position of the movable mirror for successive coincidences (or anticoincidence). Then

$$\Delta\lambda = \frac{\lambda^2}{2d}$$

where λ= 5893 Å and $\Delta\lambda$ is the doublet separation. Estimate the error of your result and compare your final value with the accepted standard value.

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